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Construction of Second Order Slope Rotatable Designs Using Pairwise Balanced Designs

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Summary

A new method of construction of second order slope rotatable designs (SOSRDs) using pairwise balanced designs (PBD) is suggested. It is shown that the new method sometimes leads to designs with less number of design points compared to designs constructed with the help of balanced incomplete block designs (BIBD).

Key Words : Response surface designs, slope rotatability, second order slope rotatable design (SOSRD).

Introduction

Hader and Park [4] introduced slope rotatable central composite designs (SRCCDs). Victorbabu and Narasimham [5] studied in detail the conditions to be satisfied by a general second order slope rotatable designs (SOSRDs). They also gave a method of construction of SOSRD using BIBD and showed that this method sometimes leads to designs with smaller number of design points than SRCCD. For definitions and notations refer to [1] and [5].

Tyagi [4] constructed second order Rotatable designs (SORD) using pairwise balanced (PB) designs. On similar lines an attempt is made here to construct SOSRD using PB designs.

2. *New Method of Construction of SOSRD Using PB Designs*

The method of construction of SOSRD using pairwise balanced design is given in the following theorem (using the notations from [2], [3] and [4].

Theorem 2.1:

If $(v, b, r, k_1, k_2, \ldots, k_p, \lambda)$ is an equireplicate PB design and

$$k = \sup\,[k_1, k_2, \ldots, k_p],$$

$2^{t(k)}$ denotes a resolution V fractional replicate of 2^k in ± 1 levels and n_0 is the number of central points, then the design points,

$$\left[1-(v, b, r, k_1, k_2, \ldots, k_p, \lambda)\right] \times 2^{t(k)}$$

$$U\ (a, 0, 0, \ldots, 0) \times 2^1 U (n_0)$$

give a v–dimensional SOSRD in

$N = b \times 2^{t(k)} + 2v + n_0$ design points, where a^2 is a positive real root (if it exists) of the biquardratic equation,

$$(8v - 4N)a^8 + 8vr \times 2^{t(k)} a^6 + [2vr^2 \times 2^{2t(k)} + \{((12 - 2v)\lambda - 4r) N + (16\lambda - 20v\lambda + 4vr)\} \times 2^{t(k)}] a^4 + [4vr^2 + (16 - 20v) r\lambda] \times 2^{2t(k)} a^2 + [(5v - 9)\lambda^2 + (6 - v) r\lambda - r^2] N \times 2^{2t(k)} + (vr + 4\lambda - 5v\lambda) r^2 \times 2^{3t(k)} = 0 \qquad (2.1)$$

Proof. Follows by verifying the conditions to be satisfied by an SOSRD [1][5].

Corollary. If $k_1 = k_2 = \ldots = k_p = k$, then Theorem 2.1 reduces to the method of construction of SOSRD through BIBD.

Example. We illustrate the use of Theorem 2.1 by constructing a SOSRD for 6–factors with the help of the PB design (v=6, b=7, r=3, k_1=3, k_2=2, λ=1).

The design points,

$$[1 - (6, 7, 3, 3, 2, 1)] \times 2^3\ U (a, 0, 0, \ldots, 0) \times 2^1 U (n_0 = 1)$$

will give a SOSRD in N = 69 design points for six factors.

The biquadratic equation 2.1 in a^2 for this case is

$$228a^8 - 1152a^6 - 32a^4 + 6144a^2 - 16128 = 0 \qquad (2.2)$$

(2.2) has only one positive real root $a^2 = 4.5314$.

It may be pointed out here that this SOSRD has only 69 design points for 6–factors, where as the corresponding SOSRD obtained through a BIB design (v=6, b=15, r=5, k=2, λ=1) needs 73 design points. Thus the new method sometimes leads to SOSRDs with lesser number of design points than the SOSRDs obtained through BIB designs.

The Appendix at the end gives the appropriate slope rotatability values of the parameter 'a' for designs using a PB design and star points and for different number of central points for $6 \leq \upsilon \leq 15$.

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APPENDIX
VALUES OF a FOR SOSRD USING PBD FOR $6 \le v \le 15$

[These are SOSRDs with design points $[1-(v, b, r, k_1, k_2, \ldots, k_p, \lambda)] \times 2^{t(k)}$
$V(a, 0, \ldots, 0) \times 2^1 \; v\,(n_0)]$

(6, 7, 3, 3, 2, 1)			(8, 15, 6, 4, 3, 2, 2)			(9, 15, 6, 4, 3, 2)		
n_0	N	a	n_0	N	a	n0	N	a
0	68	2.164	0	256	2.716	0	258	2.775
1	69	2.129	1	257	2.707	1	259	2.764
5	73	2.004	5	261	2.672	5	263	2.720
10	78	1.890	10	266	2.634	10	268	2.670
15	83	1.817	15	271	2.600	15	273	2.627
20	88	1.770	20	276	2.571	20	278	2.590
25	93	1.739	25	281	2.545	25	283	2.559

(9, 11, 5, 5, 4, 3, 2)			(10, 11, 5, 5, 4, 2)			(12, 16, 6, 6, 5, 4, 3, 2)		
n_0	N	a	n_0	N	a	n0	N	a
0	194	2.853	0	196	2.911	0	536	3.112
1	195	2.839	1	197	2.893	1	537	3.106
5	199	2.786	5	201	2.825	5	541	3.083
10	204	2.731	10	206	2.755	10	546	3.057
15	209	2.687	15	211	2.700	15	551	3.033
20	214	2.653	20	216	2.659	20	556	3.013
25	219	2.625	25	221	2.628	25	561	3.994
(13, 16, 6, 6, 5, 4, 3, 2)			(13, 15, 7, 7, 6, 5, 3)			(14, 16, 6, 6, 5, 4, 2)		
n_0	N	a	n_0	N	a	n0	N	a
0	538	3.149	0	896	4.592	0	540	3.194
1	539	3.142	1	897	4.586	1	541	3.185
5	543	3.114	5	901	4.564	5	545	3.150
10	548	3.082	10	906	4.536	10	550	3.111
15	553	3.053	15	911	4.509	15	555	3.077
20	558	3.028	20	916	4.482	20	560	3.046
25	653	3.006	25	921	4.457	25	565	3.020

(14, 15, 7, 7, 6, 3)			(15, 16, 6, 6, 5, 2)		
n_0	N	a	n_0	N	a
0	988	4.231	0	542	3.247
1	989	4.228	1	543	3.236
5	993	4.216	5	547	3.194
10	998	4.201	10	552	3.146
15	1003	4.188	15	557	3.014
20	1008	4.176	20	562	3.067
25	1013	4.165	25	567	3.035